A MORTAR FINITE ELEMENT METHOD FOR FLUID-STRUCTURE INTERACTION PROBLEMS

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We investigate the interaction between a viscous incompressible fluid and a structure whose deformation defines the interface between the two. Direct numerical solution of the highly nonlinear equations governing even the most simplified models of fluid-structure interaction requires that both the flow field and the domain shape be determined as part of the solution since neither is known *a priori*. To accomplish this, previous algorithms have decoupled the solid and fluid mechanics, solving for each separately and converging iteratively to a solution which satisfies both. However, such methods may require restrictive conditions on the mesh used in order to ensure stability. An implicit scheme for which the geometry of the interface is extrapolated from a prior iteration and then used to balance the interface conditions between the fluid and the solid, thus allowing for a coupling of the two, has been shown to be stable by Grandmont, et al. [1].

In this talk, we describe a non-conforming *hp* finite element method (adapted from [2, 3]) which solves the problem simultaneously on each subdomain. Mortar finite elements are used to construct approximate solutions of the corresponding partial differential equations on the fluid and structure domains as well as the flexible boundary between them. The local approximation within each subdomain is designed using stable *hp* finite elements, where both mesh refinement and degree enhancement are combined to increase accuracy. Moreover, an arbitrary Lagrangian-Eulerian (ALE) formulation is used to automatically discretize the fluid domain. This approach has been shown [4, 5] to be an effective method for fluids subject to transient dynamic loading as well as an easier treatment of fluid-structure interfaces. In particular, the grid velocities in our fluid mesh correspond to the velocity of the moving boundary in order to avoid excessive distortions of the computational mesh.

References

[1] C. Grandmont, V. Guimet, and Y. Maday, "Numerical Analysis of Some Decoupling Techniques for the Approximation of the Unsteady Fluid-Structure Interaction," *Mathematical Models and Methods in Applied Sciences*, v.11, n.8, pp.1349-1377, 2001.

[2] L. Chilton and P. Seshaiyer, "The *hp* Mortar Domain Decomposition Method for Problems in Fluid Mechanics," *International Journal for Numerical Methods in Fluids*, v.40, pp.1561-1570, 2002.

[3] F. Ben Belgacem, P. Seshaiyer, and M. Suri, "Optimal Convergence Rates of *hp* Mortar Finite Element Methods for Second-Order Elliptic Problems," *RAIRO Mathematical Modelling and Numerical Analysis*, v.34, pp.591-608, 2000.

[4] J. Donea, S. Giuliani, and J.P. Halleux, "An Arbitrary Lagrangian-Eulerian Finite Element Method for Transient Dynamic Fluid-Structure Interactions," *Computer Methods in Applied Mechanics and Engineering*, v.33, pp.689-723, 1982.

[5] T.J.R. Hughes, W.K. Liu, and T.K. Zimmermann, "Lagrangian-Eulerian Finite Element Formulation for Incompressible Viscous Flows," *Computer Methods in Applied Mechanics and Engineering*, v.29, pp.329-349, 1981.